Hyperbolic geometry

from a topologist's point of view

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#	Topic	Speaker	Date
1	Preliminaries on Differential Geometry	Andrea Schätzl	24.4.2017
-	Tag der Arbeit		1.5.2017
2	Models of hyperbolic space	Selen Cetin	8.5.2017
3	Compactification and isometries of hyperbolic space	Julian Hannes	15.5.2017
4	Hyperbolic manifolds	Stefan Wolf	22.5.2017
5	Selberg Lemma and residually finiteness	Victor Lisinski	29.5.2017
-	Pfingsten		5.6.2017
6	Elementary groups	Victor Lisinki and Gerrit Herrmann	12.6.2017
7	Margulis Lemma	Johannes Witzig	19.6.2017
8	Application of Margulis Lemma	Gerrit Herrmann	26.6.2017
9	Extensions of homotopies	Georg Lipp	3.7.2017
10	Mostow rigidity I	Roman Schießl	10.7.2017
11	Mostow rigidity II	Roman Schießl	17.7.2017
12	Hyperbolic manifolds in dimension 3		24.7.2017

Preliminaries on Differential Geometry In this talk on should recall some basic notions from differential geometry. These include Riemannian metric, sectional curvature, connection, geodesics, injectivity radius.

Models of the hyperbolic space Choose two models of the hyperbolic space and describe the isometries and geodesics in these models as it is done in [Ma16, Chapter 2.1].

Compactification and isometries of the hypberbolic space Define the end of a hyperbolic space and its compactification. Discuss the three kinds of isometries: parabolic, hyperbolic and elliptic (see [Ma16, Section 2.2]). Moreover, one should define horospheres.

Hyperbolic manifolds The aim of this talk is to interpret hyperbolic manifolds in a more algebraic way. The speaker should cover Section 3.1.1 – 3.1.4 and 3.5.1 & 3.5.2 of [Ma16]. Moreover, the speaker should provide examples. One possible example would be to do discuss the action of $\mathrm{PSL}_2(\mathbb{Z})$ on the upperhalf plane as done in 3.1.5 or [Mo15, Example 1.3.7].

Selberg's Lemma Prove [Ra06, Theorem 7.6.7] and do Exercise 7.6.5 (Hint: this exercise is solved in the proof of Theorem 7.6.7). Interpret the result in terms of a fundamental group of a hyperbolic manifold. Use the Borel–Harish-Chandra–Theorem [Mo15, Theorem 5.1.11] to show the existence of many finite volume hyperbolic manifolds in any dimension.

Elementary groups The aim of this talk is to provide some basic building blocks of hyperbolic manifolds. Go through Section 3.2.1, 3.2.2 and present Section 4.1 of [Ma16].

Margulis lemma Introduce the definition of a nilpotent group. Discuss Exercise 1.4.4 and prove [Ma16, Proposition 1.4.5]. Then discuss Section 4.2.1–4.2.3 of [Ma16].

Application of Margulis lemma Aim of this talk is to show the thick-thin decomposition. One has to go through Section 4.2.4–4.2.5. Is there time left one should prove [Ma16, Corollary 4.3.8].

Extensions of homotopies Prove [Ma16, Theorem 5.2.1].

Mostow rigidity Define the simplicial volume and prove [Ma16, Theorem 13.3.6].

Hyperbolic manifolds in dimension 3

References

[Ma16] B. Martelli, *Introduction to Geometric Topology*, CreateSpace Independent Publishing Platform, https://arxiv.org/abs/1610.02592[v1] (2016)

[Ra06] J. Ratcliffe, *Foundations of Hyperbolic Manifolds*, Graduate Texts in Mathematics, Springer Science (2006)

[Mo15] D.W. Morris, *Introduction to Arithmetic groups*, Deductive Press, https://arxiv.org/abs/math/0106063[v6] (2015)

Further reading

[1] R. Benedetti and C. Petronio, *Lectures on hyperbolic geometry*, Universitext, Springer Verlag (1991)

[2] F. Bonahon, *Geometric structures on 3-manifolds*, Handbook of Geometric Topology, Elsevier (2002)

[3] W. Thurston, *Three-dimensional geometry and topology*, Volume I, edited by Silvio Levy, Princeton Mathematical Series, 35 (1997)

[4] W. Thurston, *Three-dimensional manifolds, Kleinian Groups and hyperbolic geometry*, Amer. Math. Society Bulletin, 6 (1982)